When Must Ideal Gas Particles Be Treated Quantum Mechanically?

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Exercise:

We determine the temperatures for which particles in an ideal gas must be treated quantum mechanically.

Solution:

According to the ideal gas law, $PV = Nk_BT$, the average interparticle separation is given by

$$\Big(\frac{V}{N}\Big)^{1/3} = \Big(\frac{k_BT}{P}\Big)^{1/3}.$$

For the particles to be quantum mechanical, it is required that their thermal de Broglie wavelength exceed this average interparticle separation

$$\Big(\frac{k_BT}{P}\Big)^{1/3} < \frac{h}{\sqrt{3mk_BT}} \implies T^{5/3} < \frac{h^2}{3m}\frac{P^{2/3}}{k^{5/3}} \implies$$
$$\boxed{T < \frac{1}{k_B}\Big(\frac{h^2}{3m}\Big)^{3/5}P^{2/5}.}$$

For helium at atmospheric pressure, this temperature is about $T_{\mathrm{He,1atm}} \approx 2.93K$.