We derive a technique for estimating the uncertainty in an maximum likelihood estimate of the parameter of a probability distribution.

(Source: MacKay – Information Theory, Inference, and Learning Algorithms.)

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**Uncertainty in MLE**

We can estimate the uncertainty for our maximum likelihood parameter estimate as follows.

Suppose that the likelihood function is approximately Gaussian around the peak value $\hat{\gamma}$ with some unknown variance $\sigma_\gamma^2$:

$$p(\{v_i\}_i|\gamma) \sim \exp\left(-\frac{(\gamma-\hat{\gamma})^2}{2\sigma_\gamma^2}\right) \implies \log\left(p(\{v_i\}_i|\gamma)\right) \sim -\frac{(\gamma-\hat{\gamma})^2}{2\sigma_\gamma^2}.$$

Then, we may estimate the unknown variance $\sigma_\gamma^2$ by equating

$$\frac{\partial^2 \log\left(p(\{v_i\}_i|\gamma)\right)}{\partial\gamma^2} = -\frac{1}{\sigma_\gamma^2}.$$