Relevance of the Simple Harmonic Oscillator

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Exercise:

We explain why the simple harmonic oscillator is a useful model in many circumstances.

Solution:

Suppose we have a system with some complicated potential energy $V(x)$, as in Figure 1.

We observe that around the local minimum x_0, $V(x)$ is approximately parabolic. Taylor expanding the potential around this point, we have

$$V(x) = V(x_0) + V'(x_0)\frac{(x-x_0)}{1!} + V''(x_0)\frac{(x-x_0)^2}{2!} + \cdots \approx$$
$$V(x_0) + \frac{1}{2}V''(x_0)(x-x_0)^2$$

where we have eliminated $V'(x_0)$, which vanishes at x_0. As long as we consider small deviations, higher order terms are negligible. Furthermore, since the force is given by $F = -V'(x)$, the constant offset $V(x_0)$ has no physical significance. So we conclude that around a local minimum, the potential energy is given by

$$\boxed{V(x) \approx \frac{1}{2}V''(x_0)(x-x_0)^2 \equiv \frac{1}{2}k(x-x_0)^2.}$$

Hence, the harmonic oscillator approximation will hold for any system near an energy minimum.

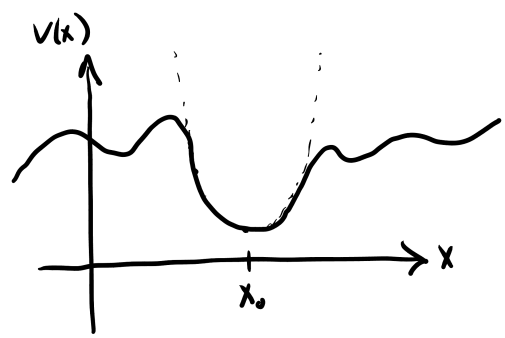

Figure 1: A complicated potential, $V(x)$. Around the local minimum x_0, $V(x)$ is approximately parabolic.