Quantum Mechanics Violates Causality

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Exercise:

We demonstrate the quantum mechanics violates causality.

Solution:

We consider the free-space time-dependent Schrödinger equation in 1D with a delta function initial condition,

$$i\hbar\Psi_t(x,t) = -\frac{\hbar^2}{2m}\Psi_{xx}(x,t) \quad , \quad \Psi(x,0) = \delta(x).$$

Taking the Fourier transform of the spatial variables in this equation gives us

$$\mathcal{F}[i\hbar\Psi_t(x,t)] = \mathcal{F}[-\frac{\hbar^2}{2m}\Psi_{xx}(x,t)] \implies i\hbar\tilde{\Psi}_t(k,t) = \frac{\hbar^2 k^2}{2m}\tilde{\Psi}(k,t) \implies$$
$$\tilde{\Psi}_t(k,t) = -\frac{i\hbar k^2}{2m}\tilde{\Psi}(k,t) \implies \tilde{\Psi}(k,t) = \tilde{\Psi}(k,0)e^{-\frac{i\hbar k^2 t}{2m}},$$

where

$$\tilde{\Psi}(k,0) = \mathcal{F}[\Psi(x,0)] = \mathcal{F}[\delta(x)] = 1.$$

Thus, the solution to the Schrödinger equation is given by

$$\tilde{\Psi}(k,t) = e^{-\frac{i\hbar k^2 t}{2m}} \implies \Psi(x,t) = \mathcal{F}^{-1}\left[e^{-\frac{i\hbar k^2 t}{2m}}\right] = \sqrt{\frac{m}{4\pi\hbar t}}\,(1-i)\,e^{\frac{imx^2}{2t\hbar}}.$$

This implies that the probability density is simply

$$\boxed{\rho(x,t) = \frac{m}{2\pi\hbar t}.}$$

This is clearly not normalizable, which is perhaps not surprising as we started with a non-normalizable initial condition. However, we observe that **the support of the probability density extends out to infinity and therefore beyond the light cone.**

Therefore, we have shown that quantum mechanics violates causality.