We derive the solution for the roots of an arbitrary quadratic function,

$$f(x) = ax^2+bx+c$$

First, we complete the square. We wish to express

$$ax^2+bx+c = k(x-\alpha)^2+d$$

Equating all derivatives, we get

$$2ax+b = 2k(x-\alpha), \quad 2a = 2k \Rightarrow k=a \Rightarrow$$

$$2ax+b = 2ax-2a\alpha \Rightarrow \alpha = -b/2a \Rightarrow$$

$$ax^2+bx+c = a\left(x^2-2x\cdot\frac{-b}{2a}+\left(\frac{b}{2a}\right)^2\right)+d$$

$\Rightarrow d = c-\dfrac{b^2}{4a}$. Thus, we have now expressed our parabola as

$$f(x) = a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}.$$

We can now find the roots by observing that this is the parabola

$$g(x) = ax^2+c-b^2/4a$$

translated left by $b/2a$. Thus, the roots of f will be the roots of g translated left by $b/2a$.

$$g(x) = 0 = ax^2 + c - b^2/4a \Rightarrow x = \pm\sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$

$$= \pm \frac{1}{2a}\sqrt{b^2 - 4ac}$$

It follows that the roots of our arbitrary parabola are given by

$$\boxed{x = \frac{-b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac}}$$

(So now we understand why this formula is true, but I still always sing the song.)