Simulating Flocks of Birds

Physics 578: Final Project

Matt Kafker

June 11, 2021

Abstract

Groups of organisms such as birds, fish, and motile bacteria are known to exhibit rich collective dynamical behavior. In this document, we present an overview of the self-propelled particle (SPP) computer simulation technique, which is useful for modeling such collective behavior. Having introduced the SPP technique, we present a detailed implementation of an SPP simulation of a flock of birds, as proposed in Bialek *et al.* (2011) [1].

Introduction

The self-propelled particle (SPP) technique allows for the simulation of the dynamics of many-body systems by discretizing Newton's laws of motion. The motion of individual particles (perhaps representing atoms, molecules, cells, insects, fish, birds, etc.) is simulated as follows.

$$\begin{aligned} \boldsymbol{v}_i(t+1) &= \boldsymbol{f}_i \\ \boldsymbol{x}_i(t+1) &= \boldsymbol{x}_i(t) + \boldsymbol{v}_i(t), \end{aligned} \tag{1}$$

where $\boldsymbol{x}_i$ and $\boldsymbol{v}_i$ are the position and velocity of particle i respectively, t is the simulation time, and $\boldsymbol{f}_i$ is some force on particle i. (Here, we have assumed without loss of generality that $\Delta t = 1$ and $m = 1$ for all particles.) Iterating (1) for each particle $i \in 1, ..., N$ allows one to construct the spatiotemporal evolution of the system of particles. As $\boldsymbol{f}_i$ is an arbitrary force, this technique is able to simulate the dynamics of a broad range of systems. We now review some basic examples.

Self-Propelled Particle Simulations

Hard Spheres

We first consider the hard sphere system, whose equations of motion are given by

$$\begin{aligned} \boldsymbol{v}_i(t+1) &= \begin{cases} -\infty \boldsymbol{e}_{ij} & \text{if } |\boldsymbol{x}_i - \boldsymbol{x}_j| < 2r_b \\ 0 & \text{if } |\boldsymbol{x}_i - \boldsymbol{x}_j| > 2r_b \end{cases} \\ \boldsymbol{x}_i(t+1) &= \boldsymbol{x}_i(t) + \boldsymbol{v}_i(t), \end{aligned} \tag{2}$$

Here, r_b is the radius of a sphere, and $\boldsymbol{e}_{ij}$ is the vector connecting particle i to particle j. In the hard sphere system, particles move linearly, except when they come within two radii of another particle, in which case they scatter elastically with one another. Frames from a hard sphere SPP simulation

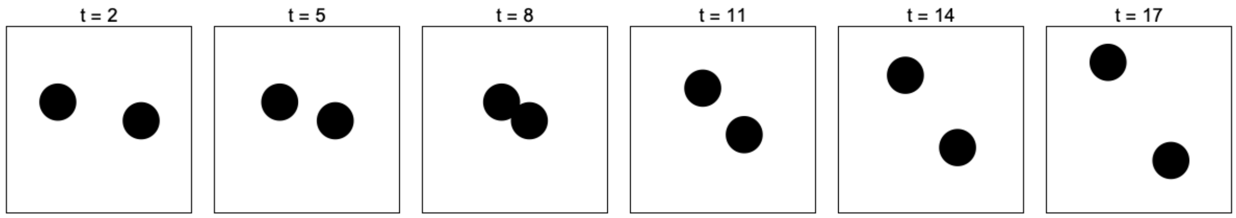

Figure 1: A glancing collision between two hard spheres.

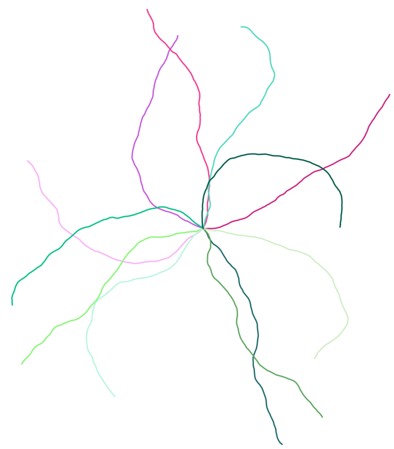

Figure 2: Persistent random walks starting from the origin. Each color indicates the trajectory of a single particle. The persistent random walk can be used to describe the motion of bacteria. Parameters of von Mises distribution: $\mu = 0$, $\kappa = 50$.

can be seen in Figure 1. By considering more particles at various densities and velocities, one can use hard sphere simulations to explore the statistical mechanics of simple liquids, as well as many other applications.

Random Walks

Another common system to model with an SPP simulation is the random walk, whose evolution is given by

$$\begin{aligned} \boldsymbol{v}_i(t+1) &= \boldsymbol{\eta}_i \\ \boldsymbol{x}_i(t+1) &= \boldsymbol{x}_i(t) + \boldsymbol{v}_i(t), \end{aligned} \tag{3}$$

where $\boldsymbol{\eta}_i$ is a random unit vector. In contrast to the deterministic linear motion with elastic scattering of the hard sphere simulation, random walks are non-interacting stochastic trajectories. If one samples $\boldsymbol{\eta}_i$ from a von Mises distribution, one produces a persistent random walk, a trajectory which can be used to model the motion of motile bacteria (see Fig. 2).

Bird Flocks

Building on equations (2) and (3), Bialek *et al.* (2011) construct an SPP simulation to model flocks of birds [1]. The equations of motion for the flock are given by

$$\begin{aligned} \boldsymbol{v}_i(t+1) &= v_0\Theta[\alpha \sum_{j\in n_c^i} \boldsymbol{v}_j(t) + \beta \sum_{j\in n_c^i} \boldsymbol{f}_{ij} + n_c\boldsymbol{\eta}_i] \\ \boldsymbol{x}_i(t+1) &= \boldsymbol{x}_i(t) + \boldsymbol{v}_i(t), \end{aligned} \tag{4}$$

where v_0 is the speed of the birds, Θ is the normalization operator for vectors, n_c^i is the set of n_c birds closest to bird i, α and β are numbers, and

$$\boldsymbol{f}_{ij} = \begin{cases} -\infty \boldsymbol{e}_{ij} & \text{if } r_{ij} < r_b \\ \frac{1}{4} \cdot \frac{r_{ij}-r_e}{r_a-r_e} \boldsymbol{e}_{ij} & \text{if } r_b < r_{ij} < r_a \\ \boldsymbol{e}_{ij} & \text{if } r_a < r_{ij} < r_0. \end{cases} \tag{5}$$

$\boldsymbol{e}_{ij}$ connects particle i to particle j; r_{ij} is the distance between particles i and j; r_b is the radius of the "bird" (really, a hard sphere); and r_a, r_e, and r_0 are other constants [1].

In this simulation, birds are modeled as hard spheres with stochastic forcing subject to two additional forces. The first, $\alpha \sum_{j\in n_c^i} \boldsymbol{v}_i(t)$, effectively averages each bird's velocity with that of its neighbors. The second, $\beta \sum_{j\in n_c^i} \boldsymbol{f}_{ij}$ encodes hard-core repulsion at very short distances, repulsion at intermediate distances, and attraction at farther distances.

Figure 3 shows several frames from a simulation a bird flock using Eq. 4.

Equation 4 exhibits a property which is crucial to the phenomenon of flocking: the non-stochastic forces apply to a bird's *topological* neighbors, n_c^i, not its metric neighbors. Indeed, it is understood that in real flocks in nature, individuals will interact with a fixed number of neighbors (~ 7), regardless of their spatial separation from those neighbors. [1] [2] Cavagna *et al.* (2014) argue that this behavior contributes to the robustness of the flock structure, which in turn has adaptive advantages for avoiding predators.

Discussion

In summary, we have reviewed the self-propelled particle computer simulation technique for modeling collective behavior in physical and biological systems. We explored the hard sphere system and the random walk, and we combined and supplemented these models to arrive at the model for flocking proposed by Bialek *et al.* (2011).

Going forward, it would be valuable to recreate some of Bialek *et al.*'s statistical analysis on the data generated from the SPP simulations. Bialek *et al.* use maximum entropy inference on data from real flocks of starlings to determine empirically the number of topological connections to neighbors and the strength of the interaction between birds. The authors apply this same inference procedure to their simulated SPP data and compare the results, showing that the assumptions underlying the SPP simulations are generally valid. [1] It would be valuable for me to learn how to apply this inference procedure to my simulation data. Moreover, if I had access to experimental data from real bird flocks, it would be valuable to apply the inference procedure to that data as well.

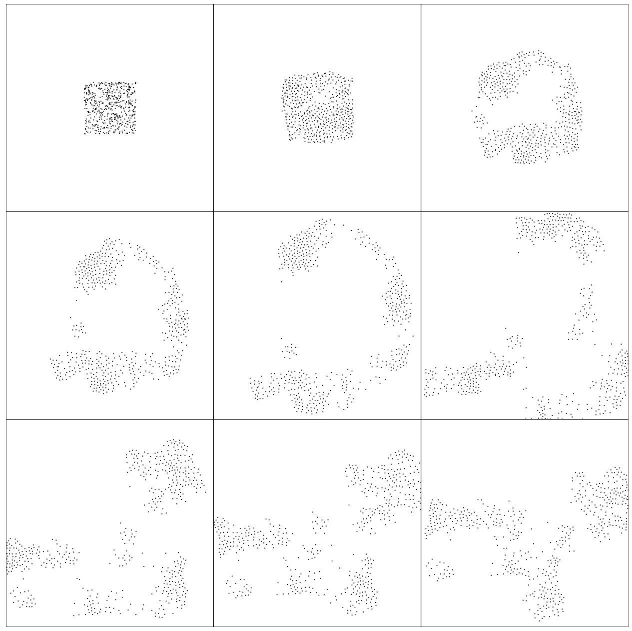

Figure 3: Frames from an SPP simulation of a bird flock in two dimensions. Birds start randomly distributed within the square with random orientations in the first frame. Frames grabbed from a simulation of 512 birds for 2500 steps in a box of size 80 with reflecting boundary conditions. Parameter values: $v_0 = 0.05, \alpha = 35, n_c = 7, \beta = 5, r_b = 0.2, r_e = 0.5, r_a = 0.8, r_0 = 1$.

As mentioned in the abstract and introduction, the number of systems in nature that one can model using SPP simulations is vast. Another valuable future direction for this work might be to use the SPP simulations and maximum entropy inference outlined in [1] and [2] to analyze collective behavior of other species, such as schools of fish or swarms of insects.

References

[1] W. Bialek, A. Cavagna, I. Giardina, T. Mora, E. Silvestri, M. Viale, and A. M. Walczak. Statistical mechanics for natural flocks of birds. *Proceedings of the National Academy of Sciences*, 109(13):4786–4791, 2012.

[2] A. Cavagna and I. Giardina. Bird flocks as condensed matter. *Annu. Rev. Condens. Matter Phys.*, 5(1):183–207, 2014.