Exercise: We show that odd moments of symmetric probability distributions vanish.

(Source: undergraduate stat mech)

Let $\rho(x)$ be a 1D continuous probability density that is symmetric under $x \mapsto -x$. Let n be an odd positive integer. Then, the nth moment of ρ

$$\langle x^n \rangle = \int_{-\infty}^{\infty} x^n \rho(x)\,dx$$

has an odd integrand under $x \mapsto -x$:

$$x^n \rho(x) \mapsto (-x)^n \rho(-x) = (-1)^n x^n \rho(x) = -x^n \rho(x).$$

Thus, the integral vanishes, and we have

$$\langle x^n \rangle = 0. \qquad \boxed{}$$