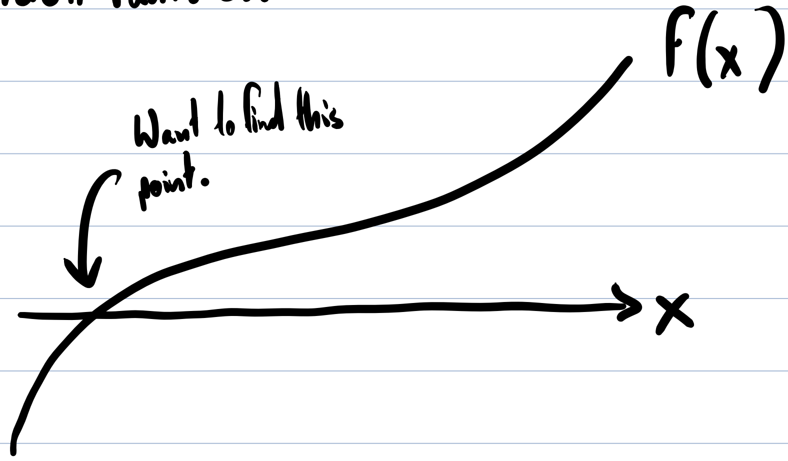
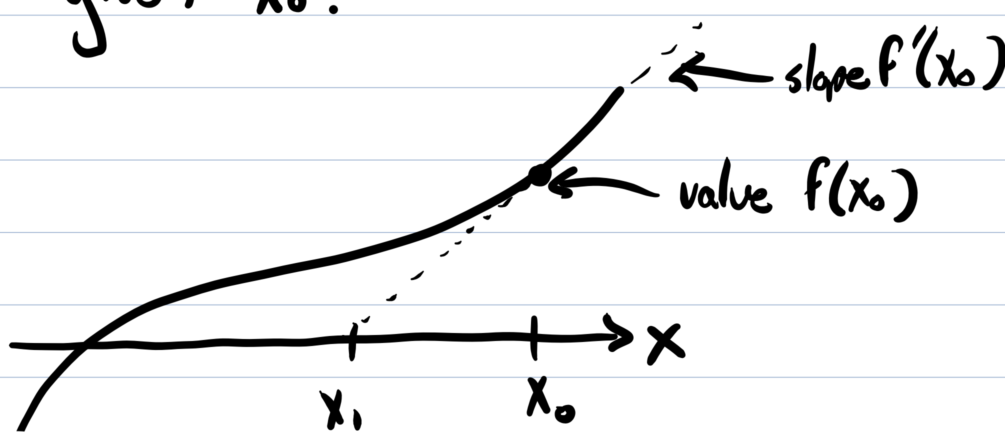
Exercise 1 We derive Newton's method for finding the roots of a differentiable function.

Suppose we have some differentiable function, and wish to find the point where it vanishes.



We proceed as follows. Pick a starting guess, x_0. Then, we update our guess to be the point where the tangent line to $f(x_0)$ intersects the axis, x_1. Then, we repeat this procedure until we approach the root. (For the present, we assume that f is sufficiently nice that this procedure will converge.)

We find x_1 given x_0.



From the diagram above, we can see that the tangent line will intersect the axis when

$$f(x_0) + f'(x_0)(x_1 - x_0) = 0 \iff$$

$$x_1 = x_0 - f(x_0)/f'(x_0).$$

We can now find x_2 given x_1, and so on. Thus, Newton's method is

- Guess x_0
- Iterate $x_n = x_n - f(x_{n-1})/f'(x_{n-1})$

  until $|f(x_n)| < \varepsilon$ for some tolerance ε.