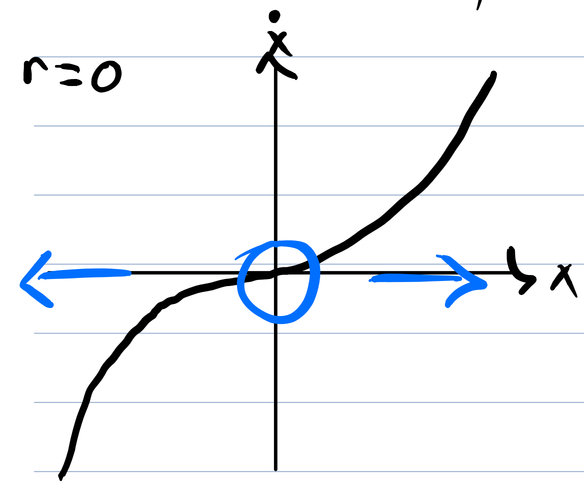
For the following systems, we sketch all vectorfields we get as r is varied. We find the value of r at which the pitchfork bifurcation occurs. We classify the bifurcation, and we plot x^* v. r.

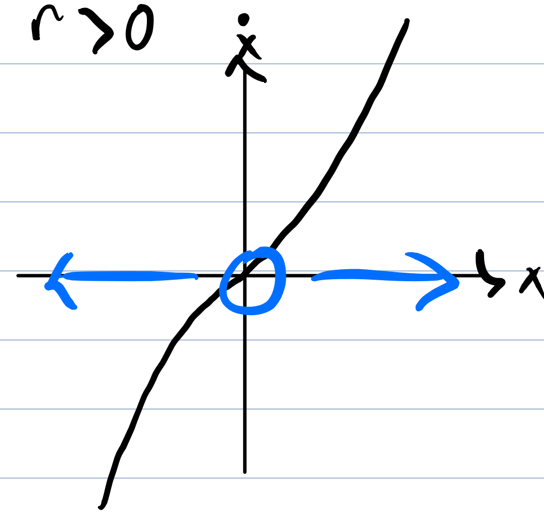
1.1

Let $\dot{x} = rx + 4x^3$. Then, we have

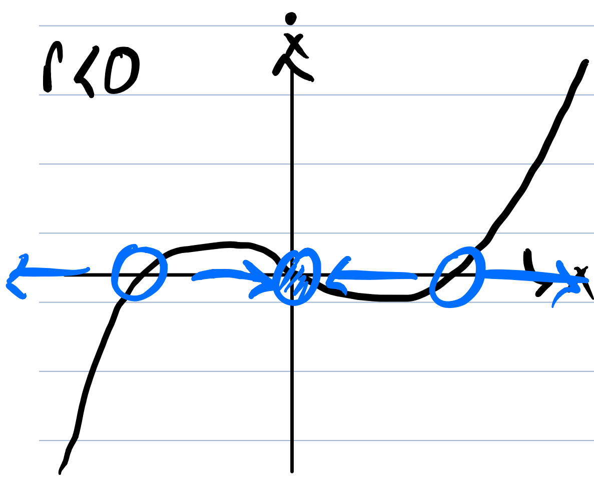
$r=0$



$r>0$



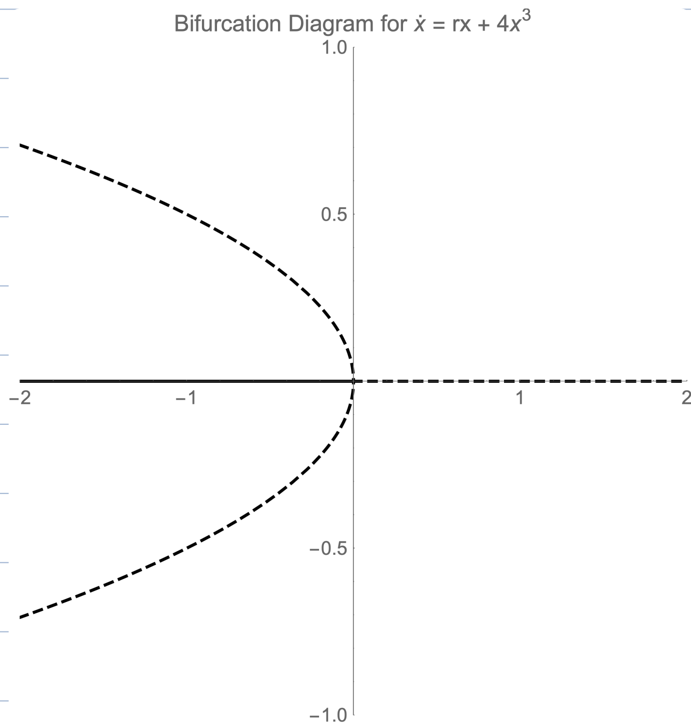
$r<0$



Thus, we observe a pitchfork bifurcation at $r=0$. By plotting the fixed points, we observe that this is a subcritical pitchfork bifurcation.

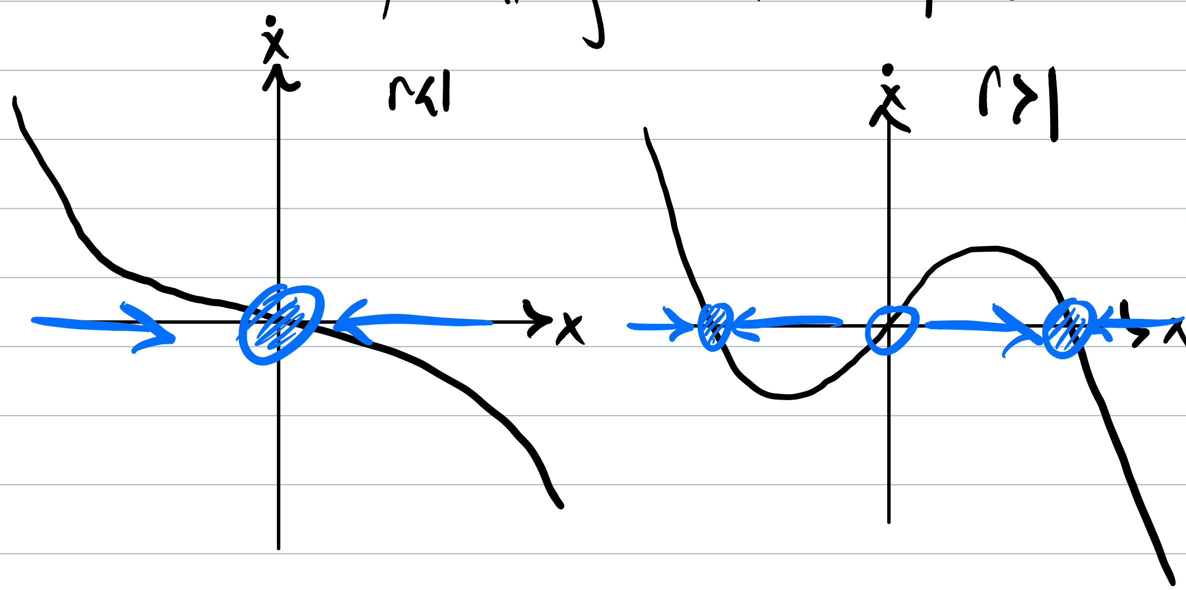
$$rx = -4x^3 \Rightarrow r = -4x^2 \Rightarrow x = \pm\frac{\sqrt{r}}{2}$$

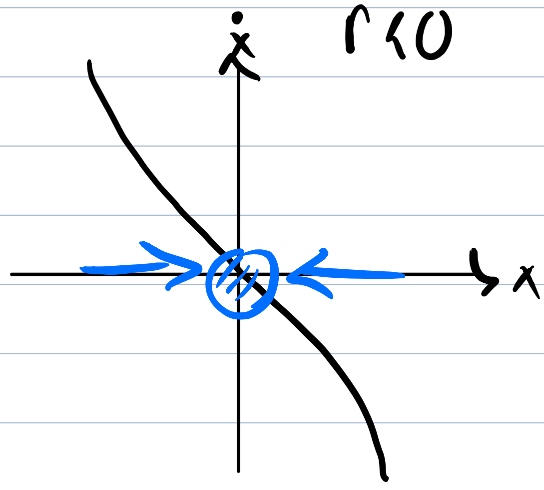
The resulting bifurcation diagram is



2.1

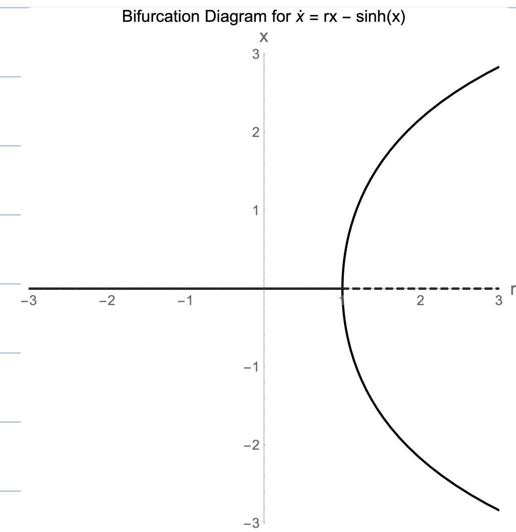
Let $\dot{x} = rx - \sinh x$. Then, the following vector fields are possible:





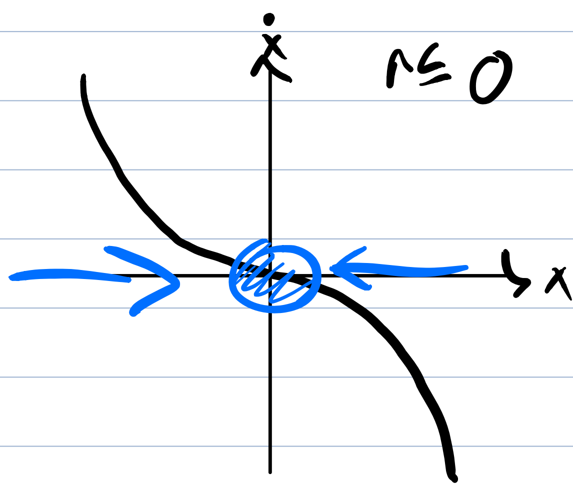
This system has a supercritical pitchfork bifurcation at $r=1$. To illustrate this, we use

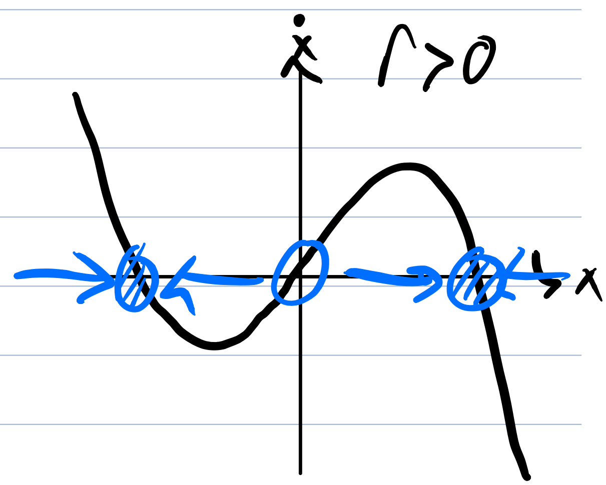
$$rx = \sinh x \Rightarrow r = \frac{\sinh x}{x}$$



3.1

Let $\dot{x} = rx - 4x^3$. Then, the following vector fields are possible:

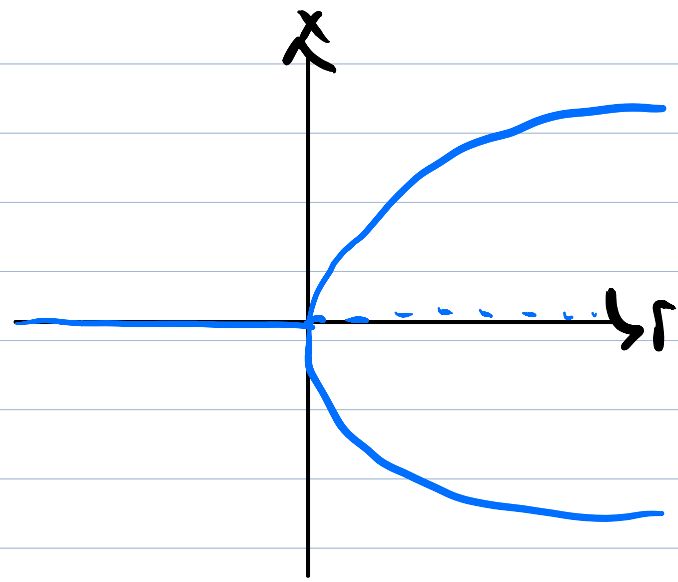




This system exhibits a supercritical pitchfork bifurcation at $r=0$.

$$rx = 4x^3 \Rightarrow r = 4x^2 \Rightarrow x = \pm\sqrt{r}/2$$

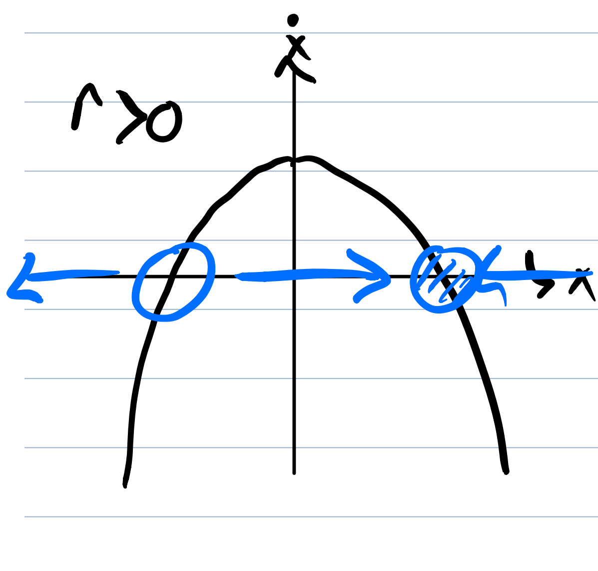
The bifurcation diagram is accordingly:

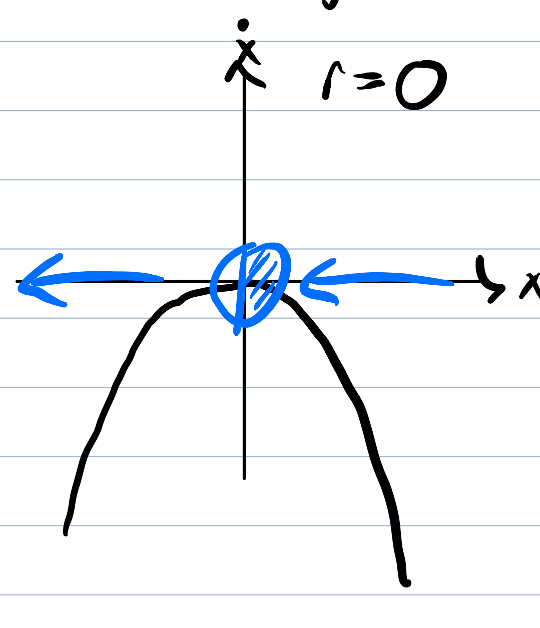


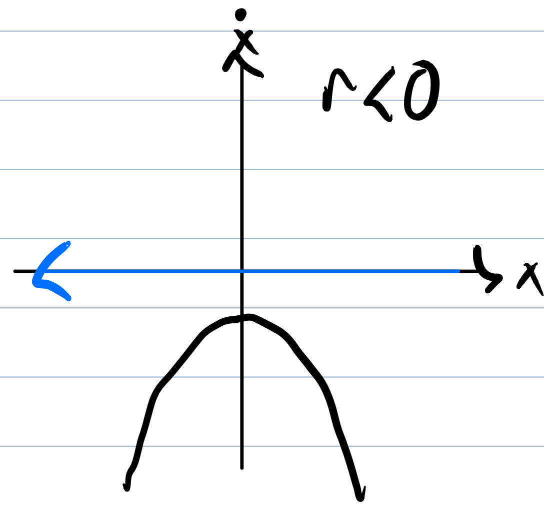
We classify the following bifurcations:

5.

$\dot{x} = r - 3x^2$. We sketch the vector fields associated with this system:



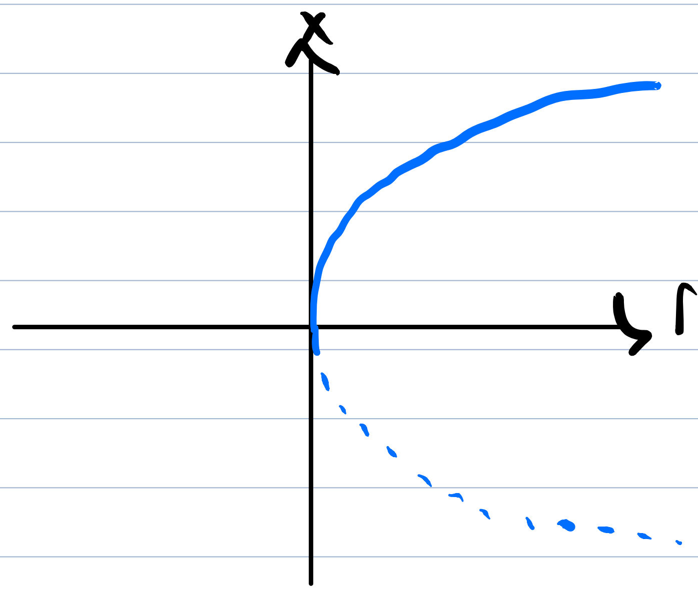




This system exhibits the appearance of two fixed points from 0 as you increase r. Thus, it has a saddle-node bifurcation at r=0.

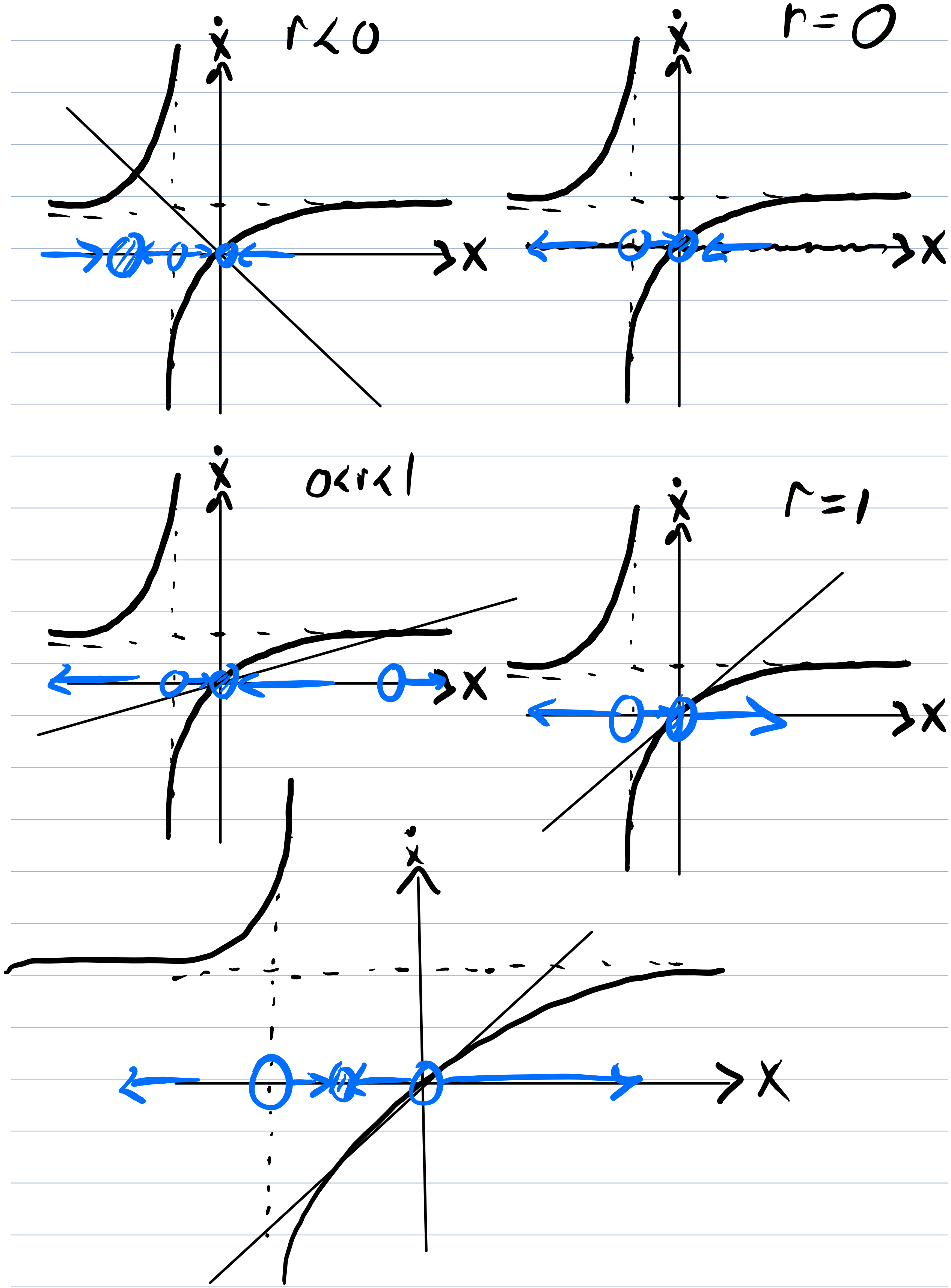
We sketch the bifurcation diagram:

$$r = 3x^2 \Rightarrow x = \pm\sqrt{r/3};$$

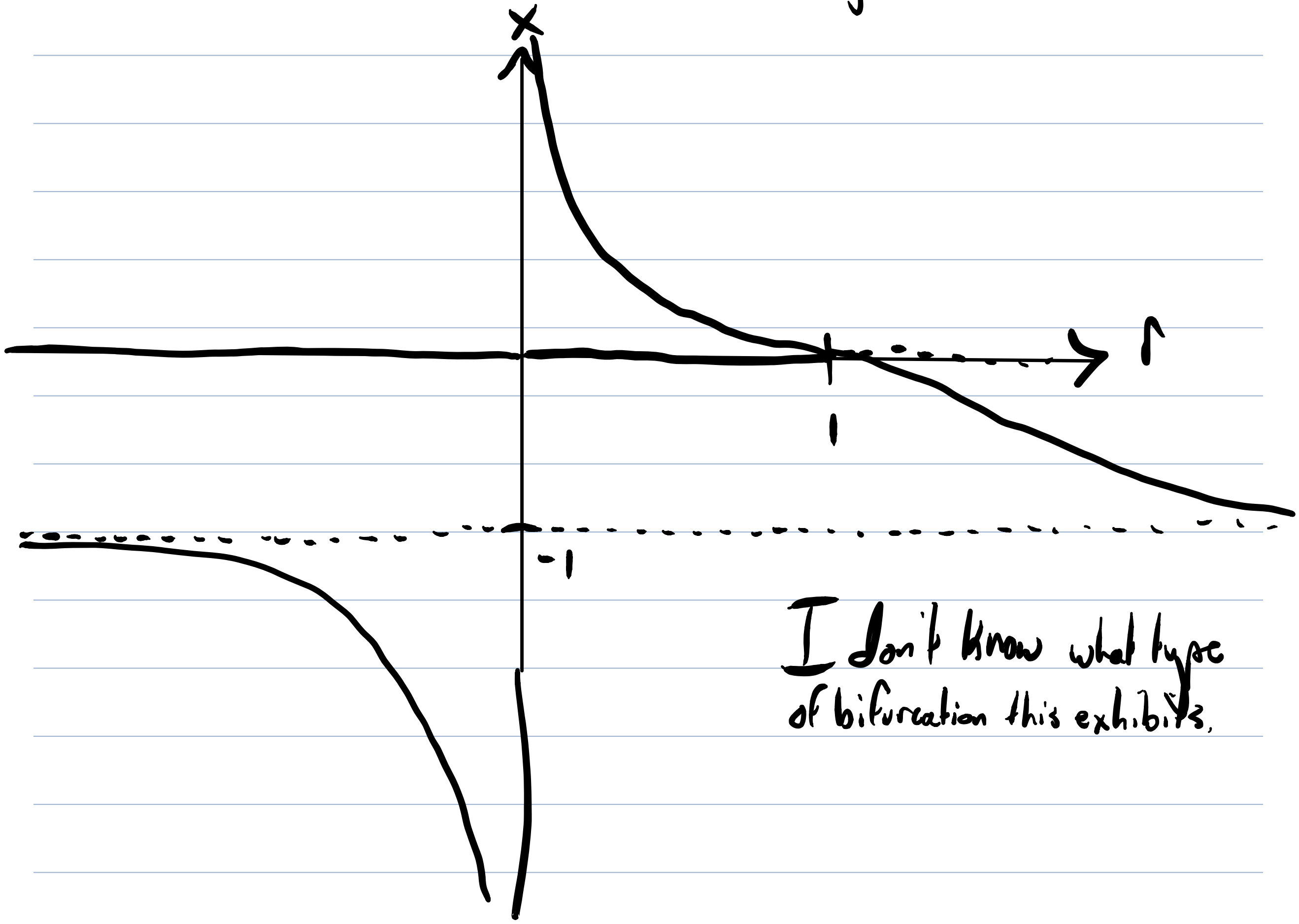


6.1

Let $\dot{x} = rx - \frac{x}{1+x}$. We sketch the vector fields associated with this system


$r<0$
$r=0$
$0<r<1$
$r=1$
$\dot{x}$
x

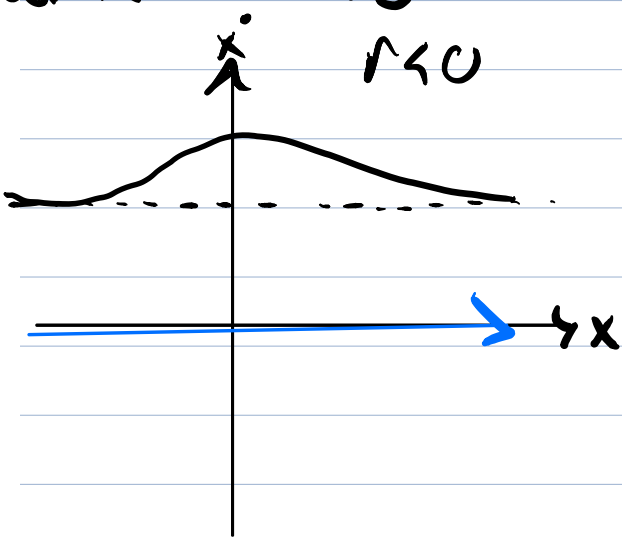
We can now construct a bifurcation diagram:

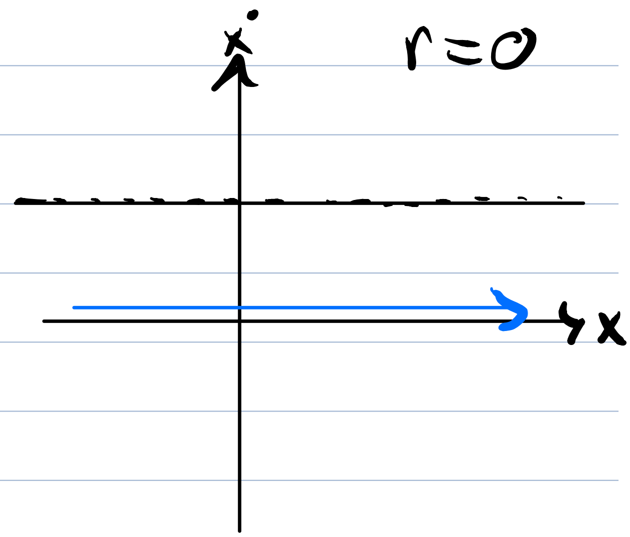


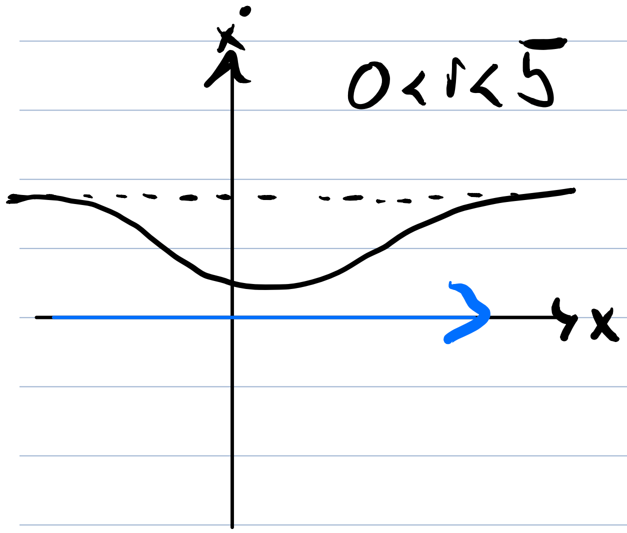
I don't know what type of bifurcation this exhibits.

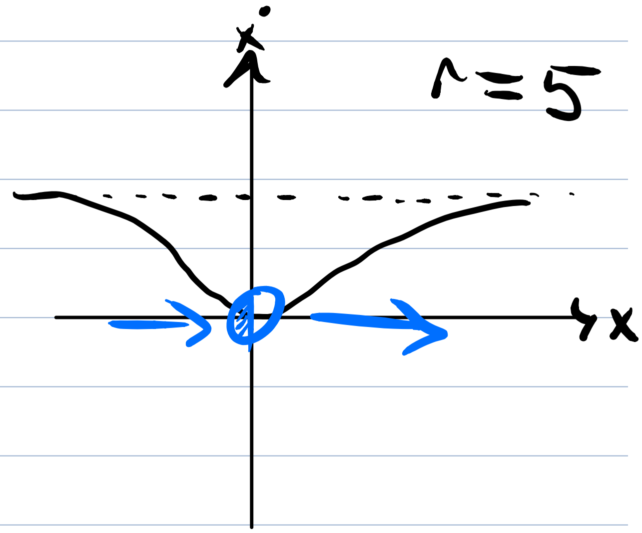
7.1

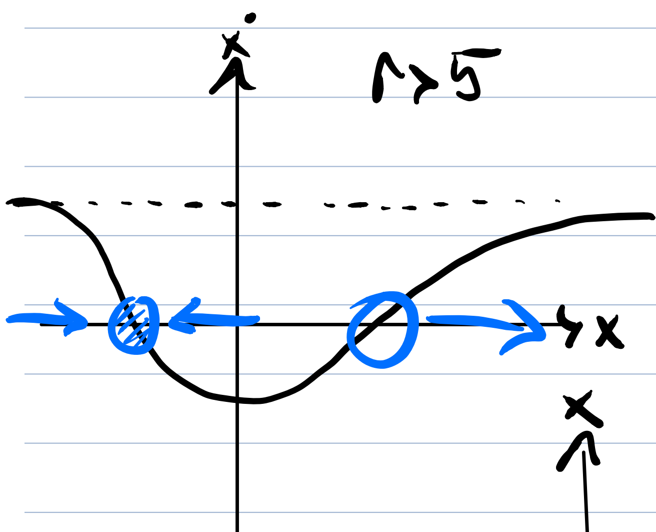
Let $\dot{x} = 5 - re^{-x^2}$. We first sketch the vector fields:





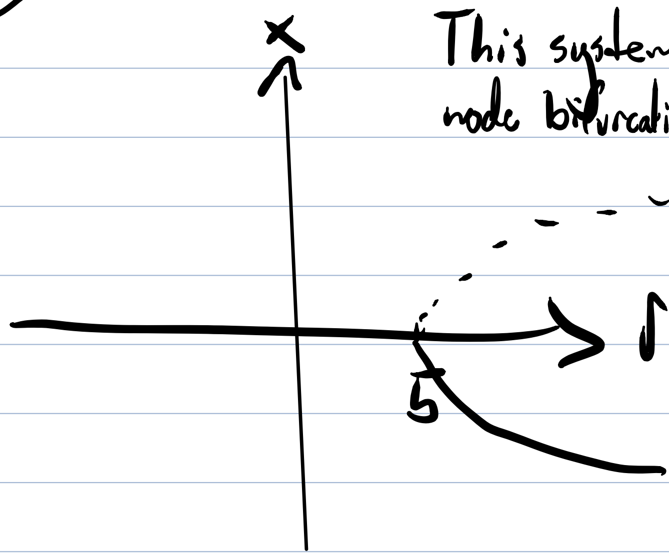






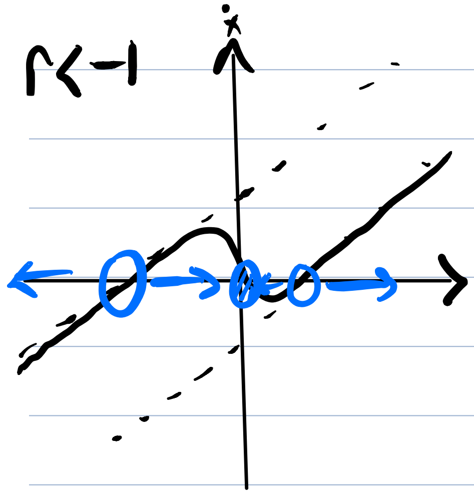
We can now construct a bifurcation diagram:

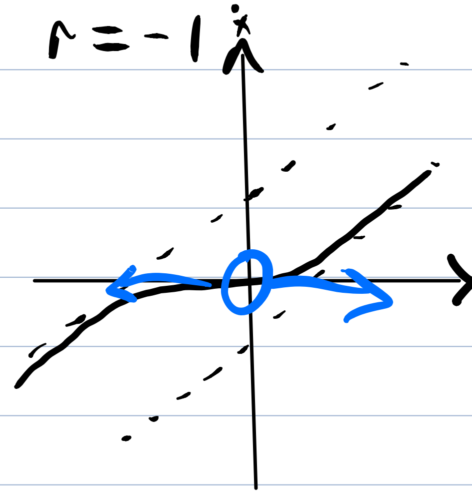
This system exhibits a saddle-node bifurcation! (At $r=5$).

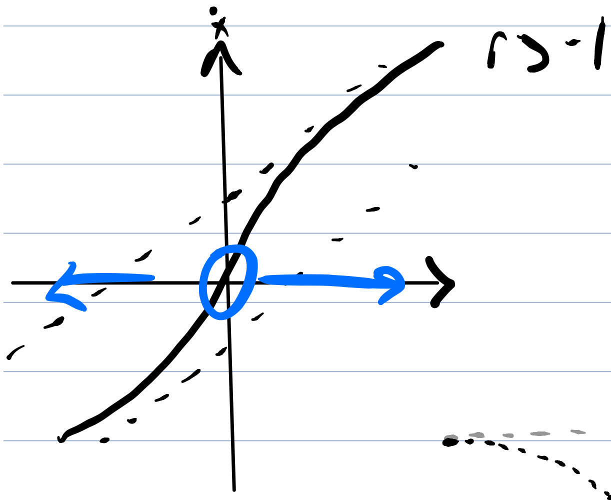


9.

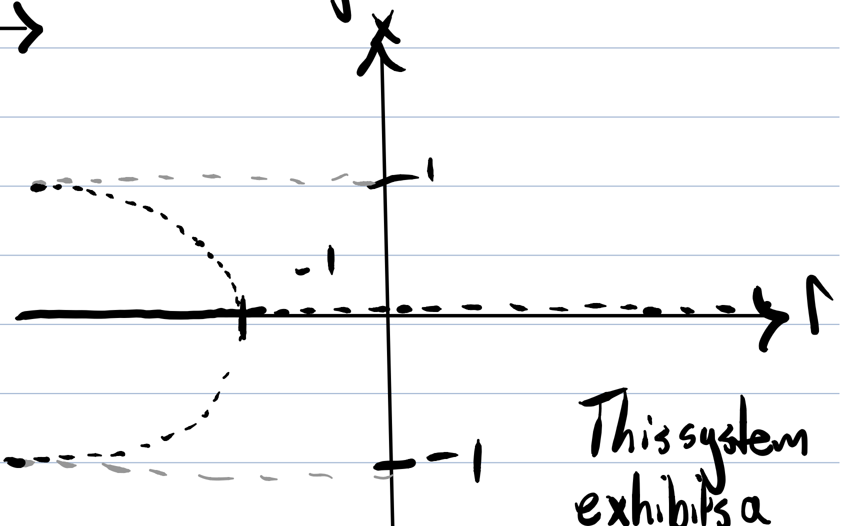
Let $\dot{x} = x + \tanh(rx)$. We draw vector fields.







We now construct a bifurcation diagram from these vectorfields.

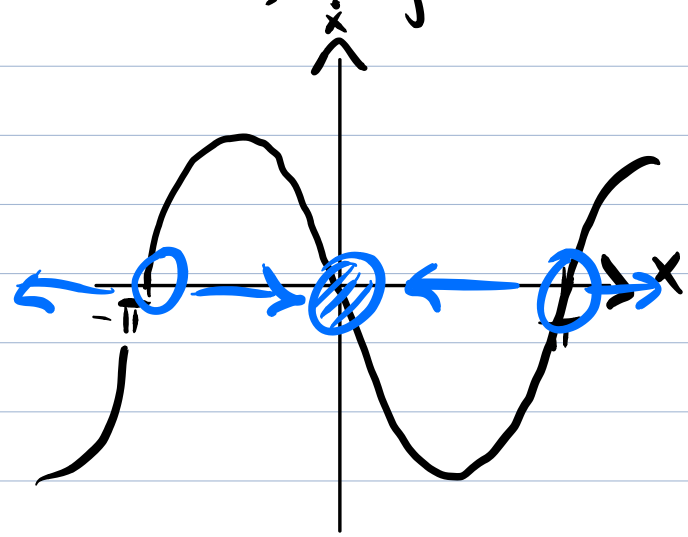


This system exhibits a subcritical pitchfork bifurcation at $r=-1$.

11.1

Let $\dot{x} = rx - \sin x$.

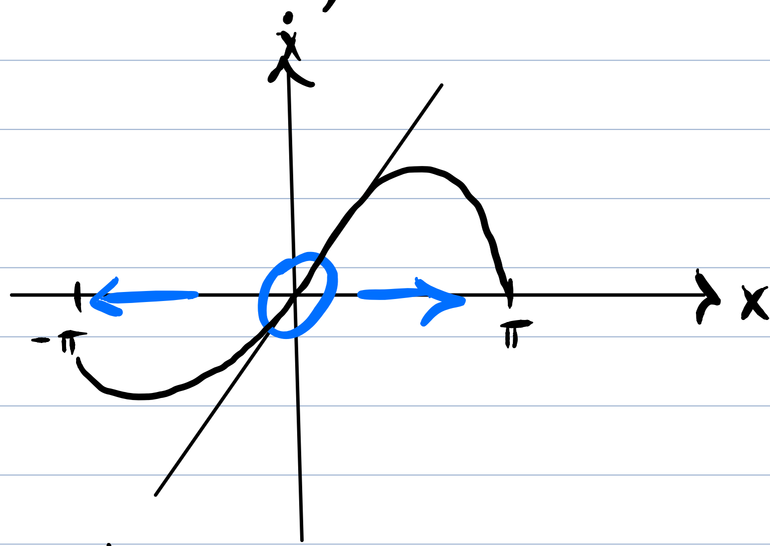
a.) When $r=0$, the system exhibits the following phase behavior:



It has stable fixed points at $2z\pi$, $z \in \mathbb{Z}$, and

unstable fixed points at $(2z+1)\pi$, $z \in \mathbb{Z}$

b.) When $r>1$, we have



one unstable fixed point at $r=0$.

c.) What happens as r decreases from ∞ to 0?

We observe that $r>1$, we have one unstable fixed point.

When $r=0$, we have an infinite number of fixed points.

When $r=1$, rx is tangent to $\sin x$

I don't know.