We skip most of the problems in this section. The central insight of the chapter is that we can treat $f(x)$ as the spatial derivative of a "potential" function, $V(x)$. That is,

$$\dot{x} = -\frac{dV}{dx}$$

Hence, when we are given $\dot{x}$, we integrate with respect to x to get $V(x)$. (Integration constant is usually set to 0.)

2.7.7

We use the existence of a potential to show that 1D systems cannot oscillate.

Proof

Let $T > 0$. Let $x(t+T) = x(t)$, and $x(t+s) \neq x(t)$ $\forall s \in (0,T)$.

Then, we consider

$$\int_t^{t+T} f(x)\dot{x}\,dt = -\int_t^{t+T} \frac{dV}{dx}\frac{dx}{dt}\,dt =$$

$$-\int_t^{t+T} \frac{dV}{dt}\,dt = V\Big|_{t+T}^{t} = V(x(t)) - V(x(t+T)) =$$

$V(x(t)) - V(x(t)) = 0$. Thus, we have $\int_t^{t+T} \dot{x}^2 \, dt = 0$

$\Rightarrow \dot{x} = 0 \Rightarrow x(t) = \text{const}.$

This contradicts the assumption that $x(t+s) \neq x(t)$.