2.5.11  
Not explore the case where asystem reades a fixed point in finite line.  
Let 
$$\dot{x} = -\dot{x}^{c}$$
,  $c \in \mathbb{R}$ ,  $x \ge 0$ .  
a) We find all C such that  $x = 0$  is a stable fixed  
point.  
We first observe that  $x = 0$  is a fixed point  $\forall c > 0$ .  
By linear stability analysis, a fixed point  $\dot{x}^{*}$  is stable provided  $f'(\dot{x}^{*})$  as  
 $f'(x) = -c x^{c-1}$   
Thus, the only case where we can use linear stability analysis is when  
 $C = 1$ . This is stable.  
More generally, the criterion for being a fixed point when  
 $\dot{x > 0}$  is that f(x) < 0 for x immediately greater than 0. This holds  
finall  $c > 0$ .

b.) Choose C such that X = 0 is stable. Can lle particle ever reach the origin in finite time? Concretely, let X(0) = 1.

We find the solution analytically:  

$$t+D = \frac{dv}{-x^{c}} = \frac{x}{c-1} \Rightarrow (t+D_{1})(c-1) = x^{1/c}$$

$$\Rightarrow x(t) = ((c-1)(t+D_{1}))^{1-c} = ((c-1)t+D)^{1-c}$$

$$x(0) = 1 = D^{1-c} \Rightarrow D = 1 \Rightarrow$$

$$x(t) = ((c-1)t+1)^{1-c}$$

$$We \text{ just want to know if it is possible for the particle to arrive of the origin. Let c<1. Then
$$\frac{1}{1-c} > 1.$$

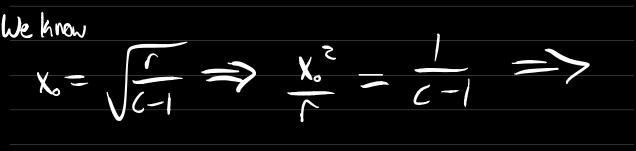
$$O = ((c-1)t+1)^{1/c-c} \Rightarrow O = (c-1)t+1 = 3$$

$$t = -\frac{1}{c-1} \text{ where } O < -\frac{1}{c-1} < \infty$$$$

We conclude that it is indeed possible for the purliche to reach its fixed point in finite time.

2.5.2 We show that the solution to  $\dot{x} = 1 + x''$  diverges in finite line from any IC. We recall that is = 1+x<sup>2</sup> has the solution X(t)=tan(t+C), which diverges when t+C= TZ. Solong as X > 1,  $1 + x^{10} > 1 + x^2$ . Hence, when X > 1, the solution from  $\dot{x} = 1 + x^{10}$  grows faster thanks solution to  $\dot{x} = 1 + x^2$ .  $1=tan(t+C) \implies t=\frac{\pi}{4}-C$ . Sowhen  $t > \frac{\pi}{4} - C$ , we know that  $\dot{x} = 1 + x^{10}$  grows harder than  $\dot{x} = 1 + x^2$ . Since Mesmaller diverges, lle larger mustas well. 2.5.3 Show that  $\dot{x} = \Gamma x + x^3$  diverges in finite live for  $\Gamma + O$ ,  $x_0 \neq 0$ .  $t+C_{l}=\int_{1\times +\infty}^{\infty} \frac{dx}{2r} \left(-\frac{1}{2\ln x} + \ln\left(r+\frac{2}{x}\right)\right) =$ 

 $\frac{1}{2r}\left(\ln\left(\frac{1}{x^2}\right) + \ln\left(r + x^2\right)\right) = \frac{1}{2r}\ln\left(\frac{r + x^2}{x^2}\right) \Longrightarrow$  $2rt + C_2 = ln(\frac{r+x^2}{x^2}) => Ce^{2rt} = l+\frac{\Gamma}{x^2}$  $\frac{\int_{0}^{2rt} - \int_{0}^{2rt} = \frac{1}{x^{2}} \implies x(t) = \frac{1}{2rt} = \frac{\int_{0}^{2rt} - \int_{0}^{2rt} \frac{1}{x^{2}}$ 





Since X. 70, this expression does not suck. Then,

 $\chi(t) = \pm \int_{\frac{r}{r}+1}^{r} e^{2rt} dt$ 

This system has a finite-time singularity when

 $\left(\frac{1}{\chi_{2}^{2}}+\right)$ ſ χ.~ n  $f + \chi_{o}^{2}$  $(+\chi_{o}^{2})$  $\frac{\chi^{2}}{1+\chi^{2}}$ n 2.5.4 We show that the IVP  $\dot{\mathbf{x}} = \mathbf{x}^{1/3}, \mathbf{x}(0) = 0$ has infinitely many solutions. This problem chearly hask trivial solution X(t)=0. Thus another solution  $X(l) = \frac{2}{3}t^{3h}$ by integration.

We next try to construct a solution that stays at X=0 until to, and then becomes non zero. Ansatz:  $\chi(t) = \Theta(t-t_0) \cdot \frac{2}{3}(t-t_0)$  $\dot{X} = \begin{cases} 0, X < t_o \\ \ell - t_o, X > t_o \end{cases}$