Hard Sphere Wall Collision Time

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Exercise:

We compute the time a hard sphere of radius R with initial configuration (x, v) will collide with the wall of a cubic box.

Solution:

Suppose the container is a cube centered at the origin with sidelength $2x_w$, such that the walls are located at $x_i = \pm x_w$ along spatial dimension i. The hard sphere will collide with the wall along the ith axis if

$$|x_i(t)| = x_w - R \implies |x_i + v_i t| = x_w - R.$$

Suppose $v_i > 0$. Then the particle will collide with the wall at $x_w - R$, and the time of the collision will be

$$x_i + v_i t = x_w - R \implies t = \frac{x_w - R - x_i}{v_i}.$$

Suppose instead $v_i < 0$. Then the particle will collide with the wall at $-(x_w - R)$, and the time of the collision will be

$$x_i + v_i t = -(x_w - R) \implies t = \frac{-(x_w - R) - x_i}{v_i}.$$

It follows that the sphere will collide with the wall along the ith axis when

$$\boxed{t_{\text{coll},i} = \frac{\text{sign}(v_i)(x_w - R) - x_i}{v_i}.}$$

The next wall collision along any axis will simply be

$$\boxed{t_{\text{coll}} = \min_i t_{\text{coll},i}.}$$