# Green's Functions for Inhomogeneous Differential Equations

Suppose we have an inhomogeneous linear differential equation

$$\hat{L}\,x(t) = f(t),$$

where $\hat{L}$ is some differential operator. Then, the "Green's function" $G(t,u)$ is defined by

$$\hat{L}\,G(t,u) = \delta(t-u).$$

The solution of the inhomogeneous equation is thus given by

$$x(t) = \int_0^\infty du\, G(t,u)\, f(u).$$

Proof

$$\hat{L}\,x(t) = \hat{L}\int_0^\infty du\, G(t,u) f(u) = \int_0^\infty du\, \hat{L}\, G(t,u)\, f(u)$$

$$= \int_0^\infty du\, \delta(t-u) f(u) = f(t).$$

Thus, if we can find the Green's function for a linear differential operator, we can solve the inhomogeneous equation by simply integrating.