We find the closed form expression for the sum

$$S_{-M,N} \quad \sum_{n=-(M-1)}^{N-1} x^n$$

It is trivial to show that $S_N = \sum_{n=0}^{N-1} x^n = \frac{1-x^N}{1-x}$.

We now find the formula for

$$S_{-M} = \sum_{n=-(M-1)}^{0} x^n = 1 + x^{-1} + \cdots + x^{-(M-1)}.$$

Consider $S_{-M} - x^{-1} S_{-M} = 1 + x^{-1} + \cdots + x^{-(M-1)} -$

$$(x^{-1} + \cdots + x^{-(M-1)} - x^{-M})$$

$$= 1 - x^{-M} = S_{-M}(1 - x^{-1}) \Rightarrow S_{-M} = \frac{1-x^{-M}}{1-x^{-1}}.$$

We combine these two expressions to get a formula for $S_{-M,N}$:

$$S_{-M,N} = \sum_{n=-(M-1)}^{N-1} x^n = S_{-M} + S_N - 1 =$$

$$\frac{1}{(1-x^{-1})(1-x)} \left[-(1-x^{-1})(1-x) + (1-x^{-M})(1-x) + (1-x^N)(1-x^{-1}) \right] =$$

$$\frac{1}{(1-x^{-1})(1-x)}\left[-(1-x^{-1})(1-x)+(1-x^{-M})(1-x)x^{-1}x+(1-x^{N})(1-x^{-1})\right]=$$

$$\frac{1}{(1-x^{-1})(1-x)}\left[-(1-x^{-1})(1-x)+(1-x^{-M})(x^{-1}-1)x+(1-x^{N})(1-x^{-1})\right]=$$

$$\frac{1}{(1-x^{-1})(1-x)}\left[-(1-x^{-1})(1-x)-(1-x^{-M})(1-x^{-1})x+(1-x^{N})(1-x^{-1})\right]=$$

$$=\frac{1}{1-x}\left(x-1-x+x^{-(M-1)}+1-x^{N}\right)=\boxed{\frac{x^{-(M-1)}-x^{N}}{1-x}}$$

We further remark that this formula holds if the starting index is positive but nonzero:

$$\sum_{n=p}^{q-1} x^n = x^p + x^{p+1} + \cdots + x^{q-1}$$

$$= S_q - S_{p+1} = \frac{1}{1-x}\left(1-x^q-(1-x^{p+1})\right) = \frac{x^{p+1}-x^q}{1-x}.$$