We show that in a system of particles obeying Newton's second law with a potential that only depends on $x^i_a - x^i_b$:

$$V = V(\{x^i_a - x^i_b \mid a, b \in \{1, \dots, N\}\})$$

total momentum is conserved.

Consider the sum

$$\sum_{a,b} \frac{\partial V}{\partial (x^i_a - x^i_b)}$$

For each pair $a \neq b$, there will be

$$\frac{\partial V}{\partial (x^i_a - x^i_b)} \quad \text{and} \quad \frac{\partial V}{\partial (x^i_b - x^i_a)} = -\frac{\partial V}{\partial (x^i_a - x^i_b)}$$

So all such pairs will cancel in the sum, and we conclude

$$\sum_{a,b} \frac{\partial V}{\partial (x^i_a - x^i_b)} = 0$$

Next, we remark that, by the chain rule,

$$\frac{\partial V}{\partial x^i_a} = \frac{\partial V}{\partial (x^i_a - x^i_b)} \frac{\partial (x^i_a - x^i_b)}{\partial x^i_a} = \frac{\partial V}{\partial (x^i_a - x^i_b)} \implies$$

$$\sum_{a,b} \frac{\partial V}{\partial (x^i_a - x^i_b)} = \sum_{a,b} \frac{\partial V}{\partial x^i_a} = N \sum_a \frac{\partial V}{\partial x^i_a} =$$

$$-N \sum_a m_a \frac{d^2 x^i_a}{dt^2} = -N \frac{d}{dt} \sum_a m_a \frac{dx^i_a}{dt} = 0$$

$$\Rightarrow \frac{d}{dt}\left(\sum_a m_a \frac{dx^i_a}{dt} \right) = 0$$

Hence, we conclude that for a system w/ such a pairwise potential, Newton's second law implies the conservation of momentum.

$\boxed{}$