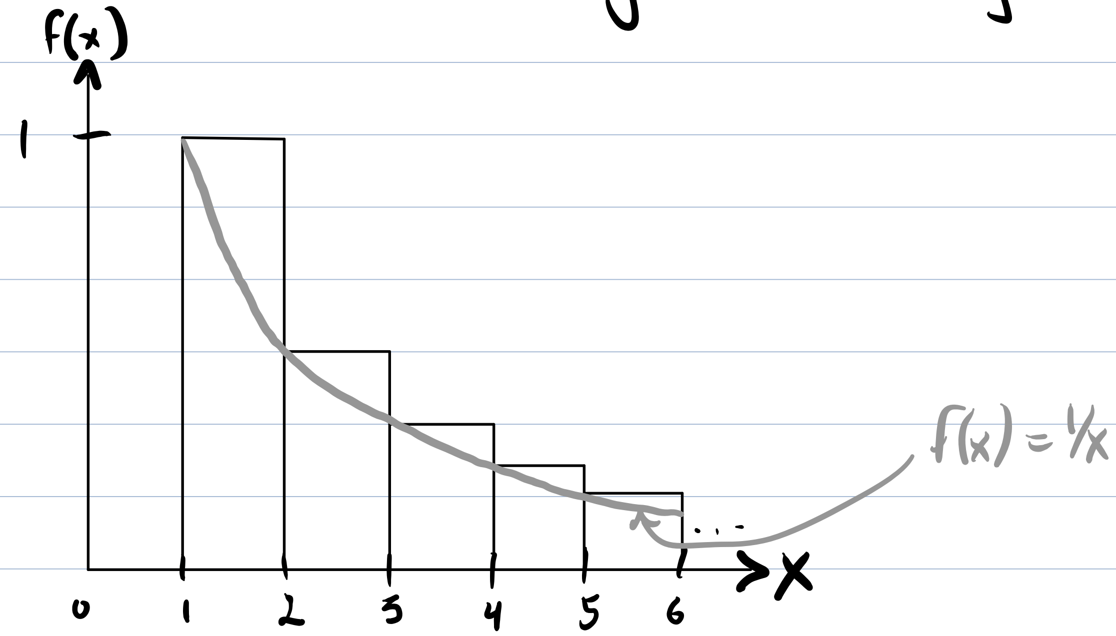
We show that the series

$$\sum_{n=1}^{\infty} 1/n$$

diverges.

<u>Proof:</u>

We observe that this sum can be understood as a Riemann sum of the function $f(x) = 1/x$, where each rectangle has a base of length 1:



That is, the total area of all the rectangles equals the value of our series. We further remark that this is a left Riemann sum, so the height of each rectangle is determined by the value of $f(x)$ at the left edge of each rectangle. Thus, it is clear that $\forall x \in [1, \infty)$, $f(x) \leq \text{rectangle}(x)$.

It follows from this observation that

$$\int_1^\infty \frac{1}{x}\,dx \leq \sum_{n=1}^\infty \frac{1}{n}.$$

(In fact, it is clear from the plot that this is a strict inequality.) However, we can evaluate this integral:

$$\int_1^\infty \frac{dx}{x} = \log x \Big|_1^\infty = \log\infty - \log 1 = \infty.$$

Thus, we have $\infty \leq \sum_{n=1}^\infty 1/n$, which implies that

$\sum_{n=1}^\infty 1/n$ diverges. $\boxtimes$