We consider a 1D spring under linear drag:

$$m\ddot{x} = -kx - \beta\dot{x} \iff \ddot{x} + \frac{\beta}{m}\dot{x} + \frac{k}{m}x = 0 \equiv$$

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = 0.$$

Guess $x = e^{zt}$. Then, we have

$$z^2 e^{zt} + \gamma z e^{zt} + \omega^2 e^{zt} = 0 \implies$$

$$z^2 + \gamma z + \omega^2 = 0 \implies$$

$$z = \frac{1}{2}\left(-\gamma \pm \sqrt{\gamma^2 - 4\omega^2}\right)$$

Suppose $\gamma^2 - 4\omega^2 > 0$. Then, our solutions are

$$\left.\begin{aligned} x(t) &= Ae^{\frac{1}{2}\left(-\gamma + \sqrt{\gamma^2 - 4\omega^2}\,t\right)} \\ x(t) &= Ae^{\frac{1}{2}\left(-\gamma - \sqrt{\gamma^2 - 4\omega^2}\right)t} \end{aligned}\right\}$$ (Recall, a second order ODE has two linearly independent solutions)

We assumed $\beta, m > 0$ (physically, drag opposes motion), so it follows that the second solution represents exponential decay. This solution decays without oscillation, and so it represents an "overdamped" oscillator.

If $\gamma^2 - 4\omega^2 < 0$, then the solution represents a decaying oscillation

$$x(t) = A e^{-\frac{\gamma}{2}t} e^{\pm\frac{1}{2} i \sqrt{4\omega^2 - \gamma^2}\, t}$$

This is an "underdamped" oscillator.

Finally, if $\gamma^2 - 4\omega^2 = 0$, then the solutions are

$$x(t) = e^{-\gamma/2\, t}, \quad t e^{-\frac{\gamma}{2}t}$$

where the factor of $t$ is required to force linear independence.