We consider a particle moving along the x axis with speed u. It is described by the 4-vector

$$x^\mu = (t, ut, 0, 0)$$

We now boost to a frame moving with velocity v, also along the x-axis, using the Lorentz transformation matrix

$$\Lambda^{\mu'}{}_{\mu} = \begin{pmatrix} \gamma & -v\gamma & 0 & 0 \\ -v\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x^{\mu'} = \Lambda^{\mu'}{}_{\mu} x^\mu = (\gamma t, -tv\gamma, 0, 0) + (-vut\gamma, ut\gamma, 0, 0)$$

$$= (\gamma t(1-uv), \gamma t(u-v), 0, 0) \Rightarrow$$

$$\left.\begin{aligned} t' &= \gamma t(1-uv) \\ x' &= \gamma t(u-v) \end{aligned}\right\} \Rightarrow \boxed{v' = \frac{x'}{t'} = \frac{u-v}{1-uv}}$$

This is the relativistic velocity formula for particles moving collinearly.