<u>Exercise</u>: We analyze the perfectly elastic collision between two point particles in one dimension, where one particle is initially stationary. We treat three limits of the ratio of the two masses.

Suppose we have two particles with masses M and m. Let mass M have initial velocity U. We solve for U' and V', the outgoing velocity of M and m, respectively.

Since the collision is elastic, momentum and kinetic energy are conserved, so we have

$$MU = MU' + mv'$$

$$\frac{1}{2}MU^2 = \frac{1}{2}MU'^2 + \frac{1}{2}mv'^2 .$$

First, we solve for v':

$$v' = \frac{M(u-u')}{m} \Rightarrow v'^2 = \frac{M(u^2-u'^2)}{m} =$$

$$\frac{M(u-u')(u+u')}{m} = v'(u+u') \Rightarrow$$

$$v' = u + u'.$$

We now treat three cases.

<u>Case 1:</u> $\frac{m}{M} \ll 1$

Momentum conservation gives

$$u = u' + \frac{m}{M} v' \approx u' \Rightarrow \boxed{\begin{cases} v' = 2u \\ u' = u \end{cases}}$$

In the limit of a very heavy incoming particle, the stationary particle acquires twice the velocity of the incoming particle.

<u>Case 2:</u> $\frac{m}{M} = 1.$

Momentum conservation gives

$u = u' + v' \iff v' = u - u'$. However, we also have

$$v' = u + u' \Rightarrow \boxed{\begin{cases} v' = u \\ u' = 0 \end{cases}}$$

For the equal mass collision, the incoming particle transfers all its velocity to the stationary particle.

Case 3: $\frac{m}{M} >> 1$

From momentum conservation, we have

$$v' = \frac{M}{m}(u - u') \approx 0 \Rightarrow \boxed{\begin{cases} v' = 0 \\ u' = -u \end{cases}}$$

If the stationary particle is very massive, the incoming particle is simply reflected.